Optimal ROAD CHARGE Level Taking in to Accounts Marginal Cost of Public Funds

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Marginal Cost of Public Funds (MCF) 1

- Marginal cost of public funds (MCF) is the marginal welfare loss of taxpayer due to the marginal tax raise
- Marginal cost of fuel tax
  (marginal excess burden)
  marginal loss of consumers divided by marginal revenue increase

Marginal Cost of Public Funds (MCF) 2

- Raise up the present tax level by 1 cent

  marginal loss of consumers (producers)

  = marginal decrease in consumers’ (producers') surplus

  = - 1cent multiplied by present level of consumption (labor supply)

  marginal revenue increase

  = 1cent * present level of consumption (labor supply)

  - (present tax level multiplied by decrease in consumption (labor supply) due to the tax up by1cent

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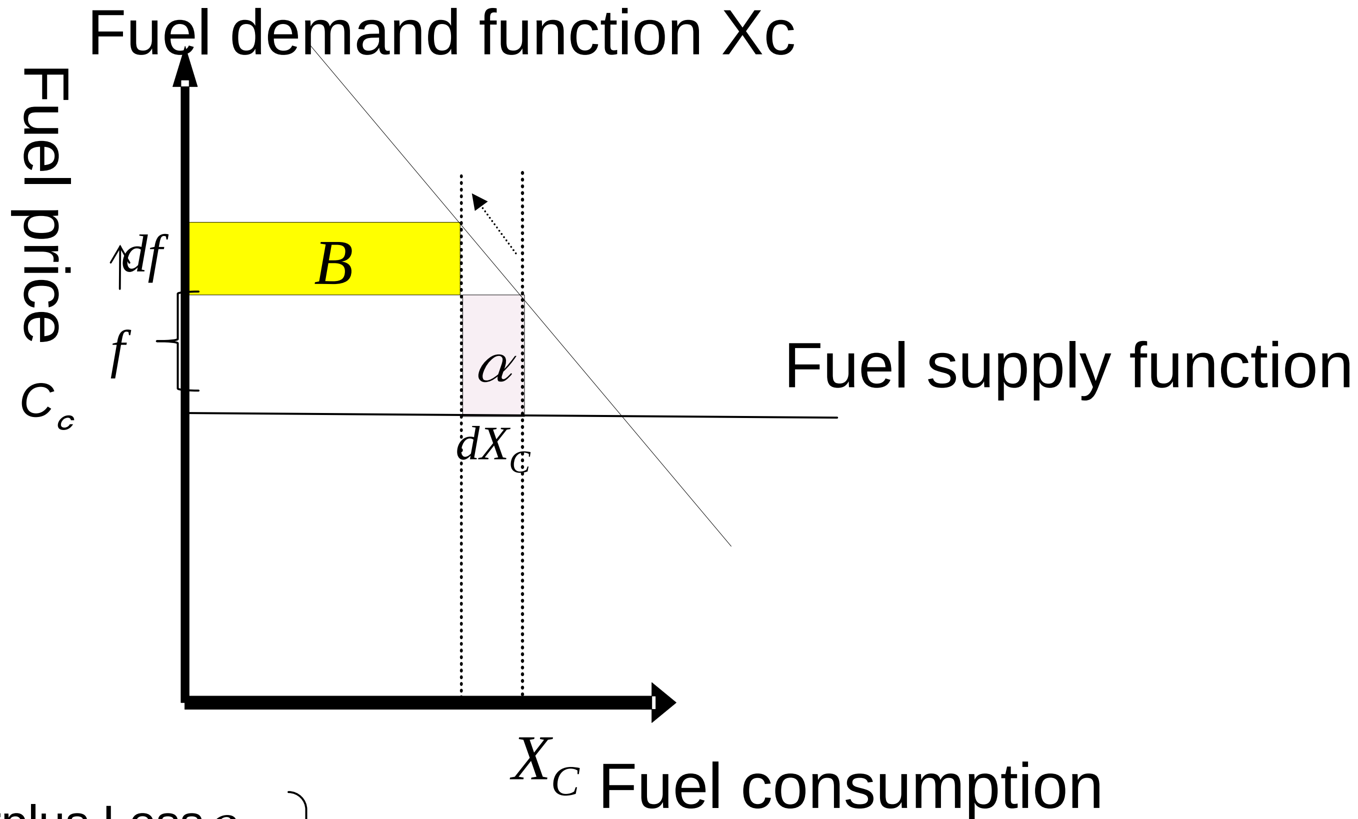
MCF of Fuel Tax



$$mcf = \frac{\left(\text{Surplus Loss}\ \beta\right)}{\left(\text{Tax net revenue}\ \beta - \alpha\right)} = \frac{-X_C df}{X_C df - f dX_C} = -\frac{-X_C}{X_C - f \frac{dX_C}{dC_c} \frac{dC_C}{df}}$$

$$= \frac{-1}{1-(f/C_C)\left|\varepsilon_C\right|}$$

MCF of Fuel Tax

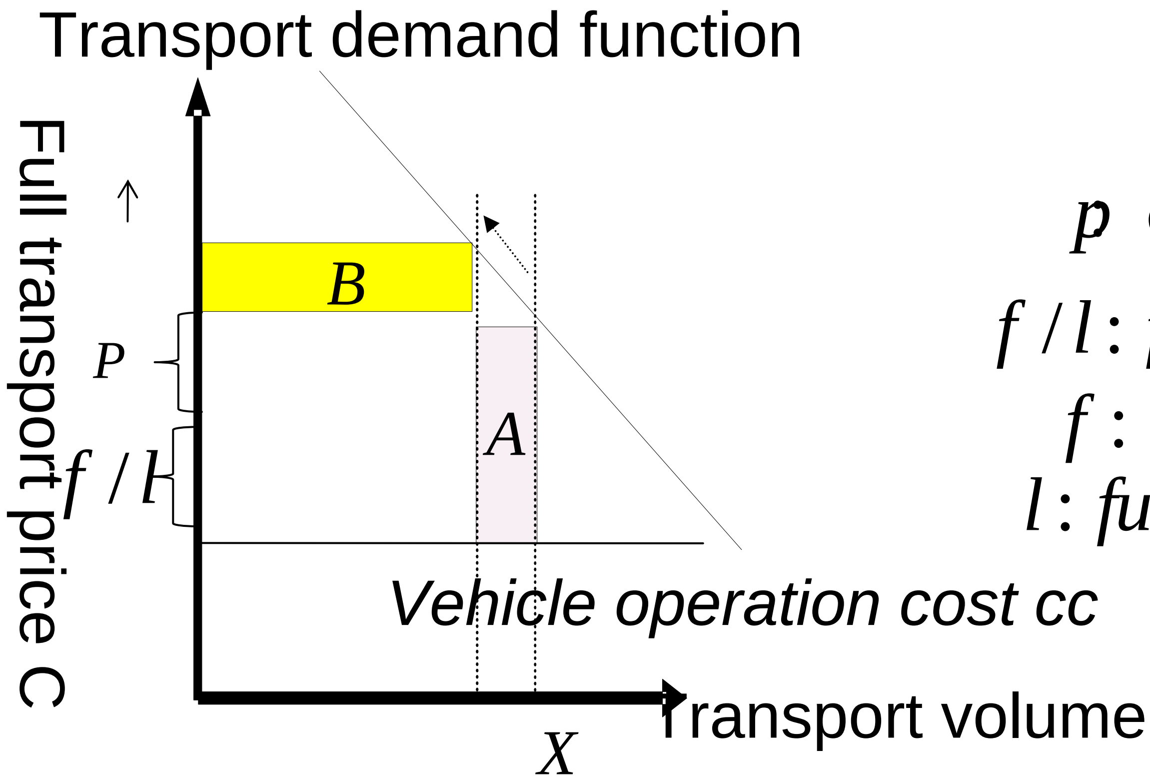
$$mcf = \frac{-1}{1-(f/C_C)\left|\varepsilon_C\right|}$$

Marginal cost of fuel tax
(marginal excess burden)

(1-(tax ratio * price elasticity))

Note absolute value of MCF is greater than 1 as tax ratio and price elasticity are usually less than 1

MCF of Road charge (mcp)



p: charge (yen/km)

f/l: fueltax (\/km)

f: fueltax (¥/l)

l: fuelefficiency (km/l)

$$mcp = \frac{\left(\text{Surplus loss} - B\right)}{\left(\text{Net revenue increase } B - A\right)} = \frac{-Xdp}{Xdp - (p + f/l)dX} = \frac{-1}{1 - |\varepsilon_p|} = \frac{-1}{1 - ((p + f/l)/C)|\varepsilon|}$$

MCF of Road charge (mcp)

- Marginal cost of Road charge (marginal excess burden)

  marginal loss of consumers / marginal net revenue increase

= -/(1-(tax ratio * full price elasticity))

Note Absolute value of MCF is greater than 1 as tax ratio and (absolute value of) full price elasticity are usually less than 1

Principle of Classical Marginal Cost Pricing

Classical marginal cost pricing says

– Price level of maximizing social surplus is

: P=0

If MCF is not taken into accounts ($mcf = 1$)

This Study says that If MCF is incorporated

– Price level of maximizing social surplus is

: P$\neq$0, where

$$-mcf \geq -1.0), say, -mcf = 1.15$$

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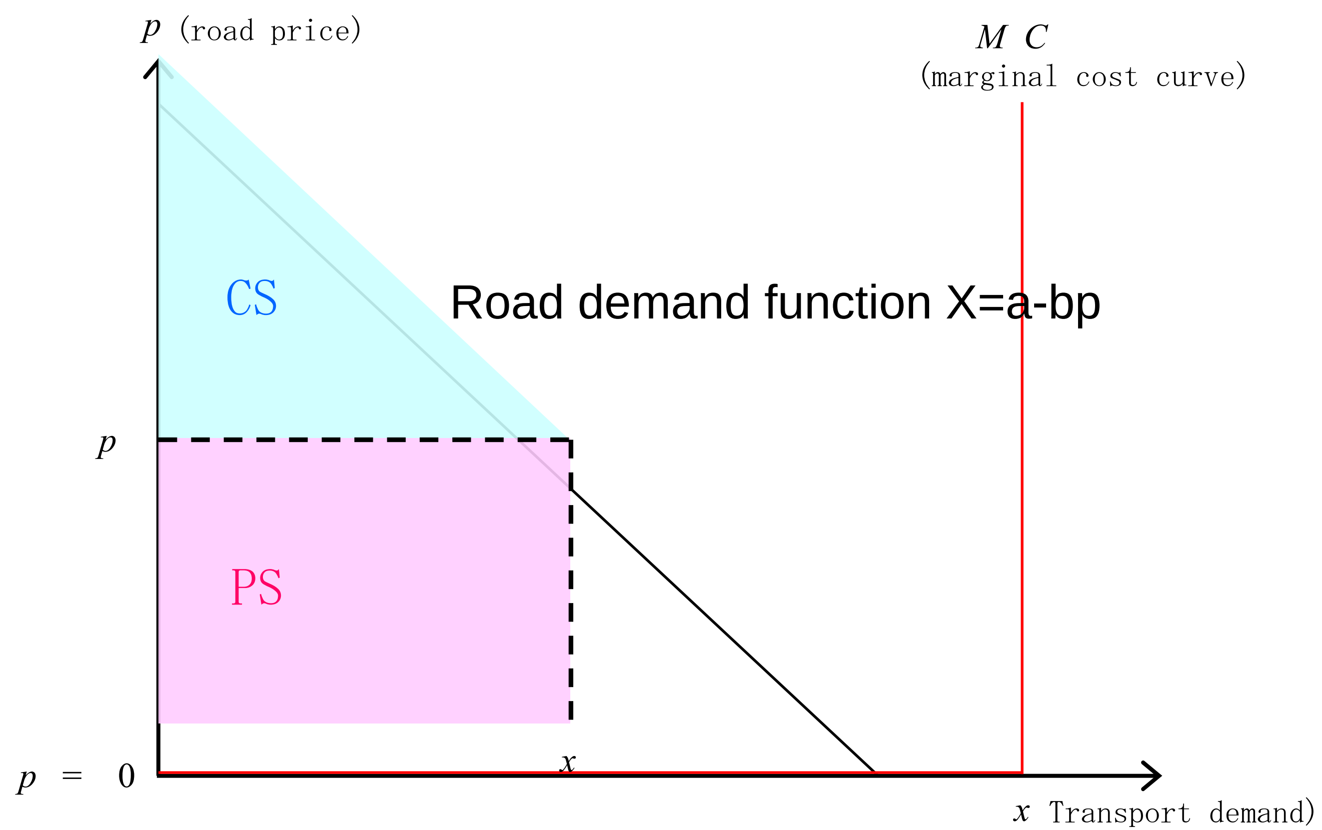
Question

If marginal cost of public funds is taken into accounts, i.e. - MCF>1.0,

why the optimal price level of maximizing the social surplus is not zero?

Social Surplus of Road Pricing Level P



Assumption: no fuel tax for simplicity

Social Surplus SS 1

- Road users’ benefits=consumers’ surplus(=Blue area of Figure of p.10)
- Road suppliers' benefits=Producers’ surplus *pX*（Pink area of Figure of p.10=Toll revenue）
  －Construction cost *C*＋Subsides *S*）　（＝0）
  f/l_H
- Assumption ：Subsidy level is such that road suppliers' benefits=0, i.e. *pX-C+S=0*, or *S=C-pX*）
- Note: Producer's surplus in case that fuel tax is imposed is *(p+f/l)* (toll revenue +fuel tax revenue)

Social Surplus SS 2

- Government raise the tax level in such a way to meet the subsidy (or reduce the expenditure of some funds to meet the subsidy).

  Government 's benefits=tax revenue *S*- subsidy *S*0

  Tax payers' disbenefits=decrease in surplus due to the tax level up= marginal cost of the tax multiplied by subsidy=$mcf \cdot S = mcf \cdot (C\text{-}pX)$

Note The beneficiary of that funds is corresponding the tax payer above in case of reducing the expenditure of some funds to meet the subsidy.

Social Surplus SS 3

- Social Surplus SS = road users' benefits + road suppliers' benefits(=0)+ government's benefits(=0)-taxpayers' disbenefits

＝consumers' surplus *CS* – MCF・subsidies

＝CS+*mcf*・(*C*-*pX*)

Social Surplus SS＝*CS*-*mcf*・*pX*+*mcf*・*C*

Note) *mcf is minus.*

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Classical Marginal Cost Pricing 1

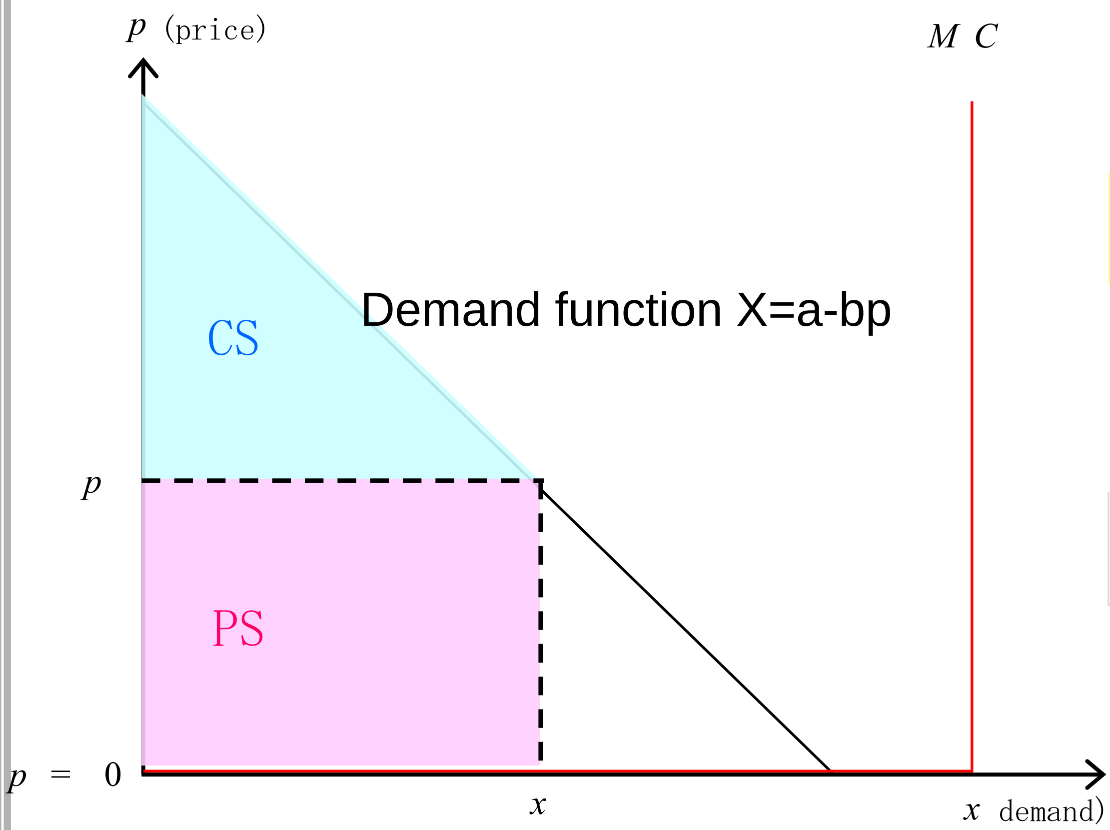
Social Surplus SS $CS\text{-}mcf \cdot pX+mcf \cdot C$

Assume mcf=-1. Then

Social Surplus SS $=CS+pX-C$

i.e. it assumes that the lump sum tax is possible and funding on it, but in reality not from the lump sum tax but excise tax.

Classical Marginal Cost Pricing 2



$$\max_{p} CS + PS = \int x\,dp + px$$

Down the level of *p*

***p*=0** maximizes *CS*+*PS* !

Take into Accounts MCF 1

$$\max SS = CS + \left(pX - C + S \right) + (S - S) + mcf \cdot S$$

$$= CS + mcf \left(C - pX \right)$$

$$= CS - mcf \cdot pX + mcf \cdot C$$

$$= CS + p \cdot (-mcf) \cdot X + const$$

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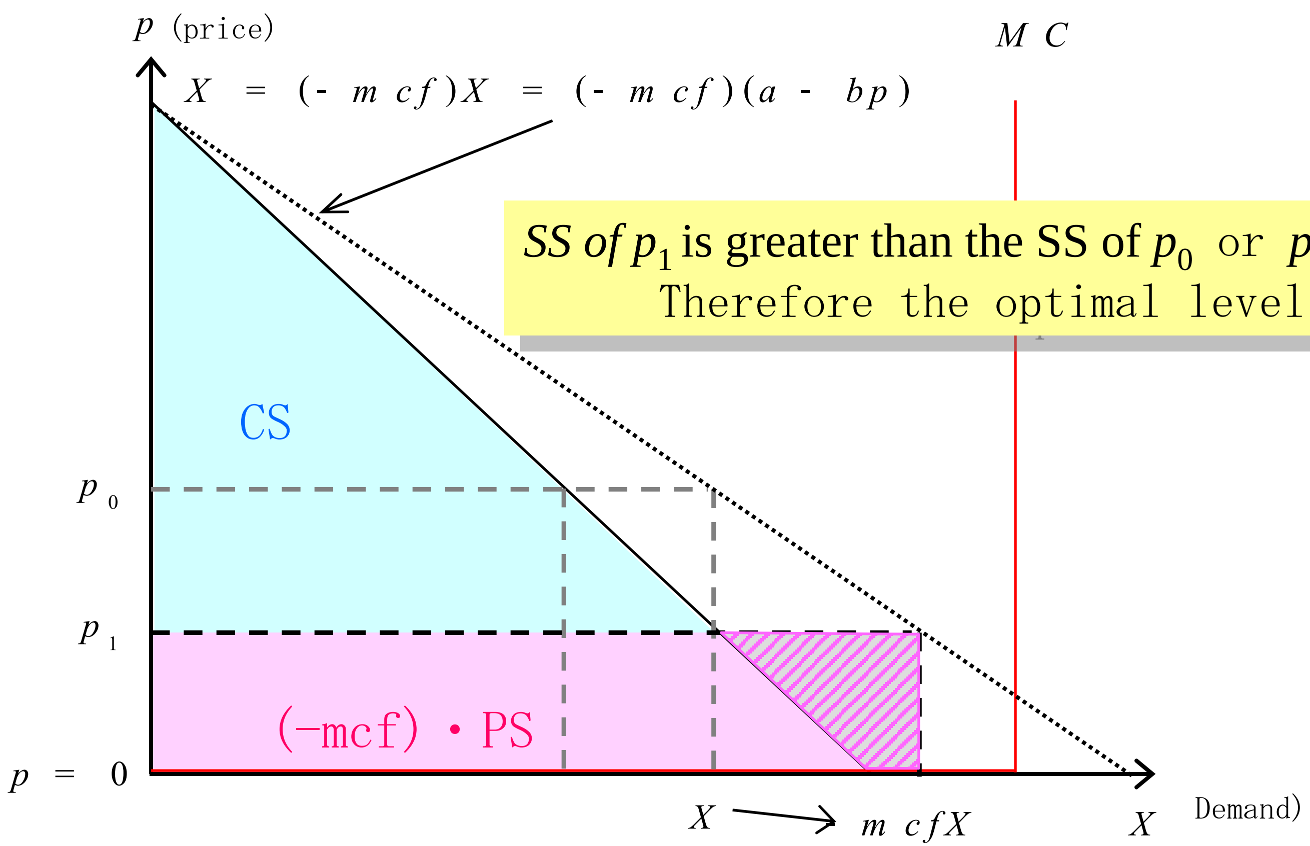
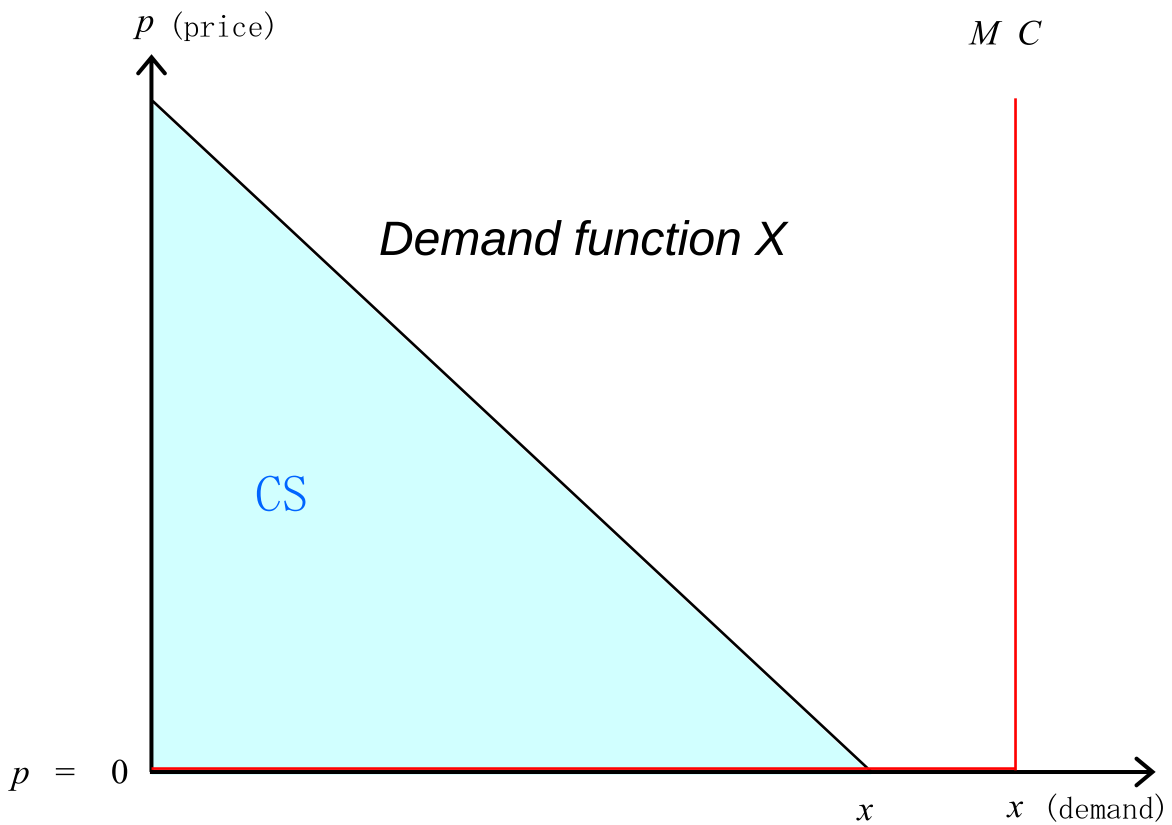
Take into Accounts MCF 2

Max $SS\ CSp \cdot (-mcf) \cdot X$



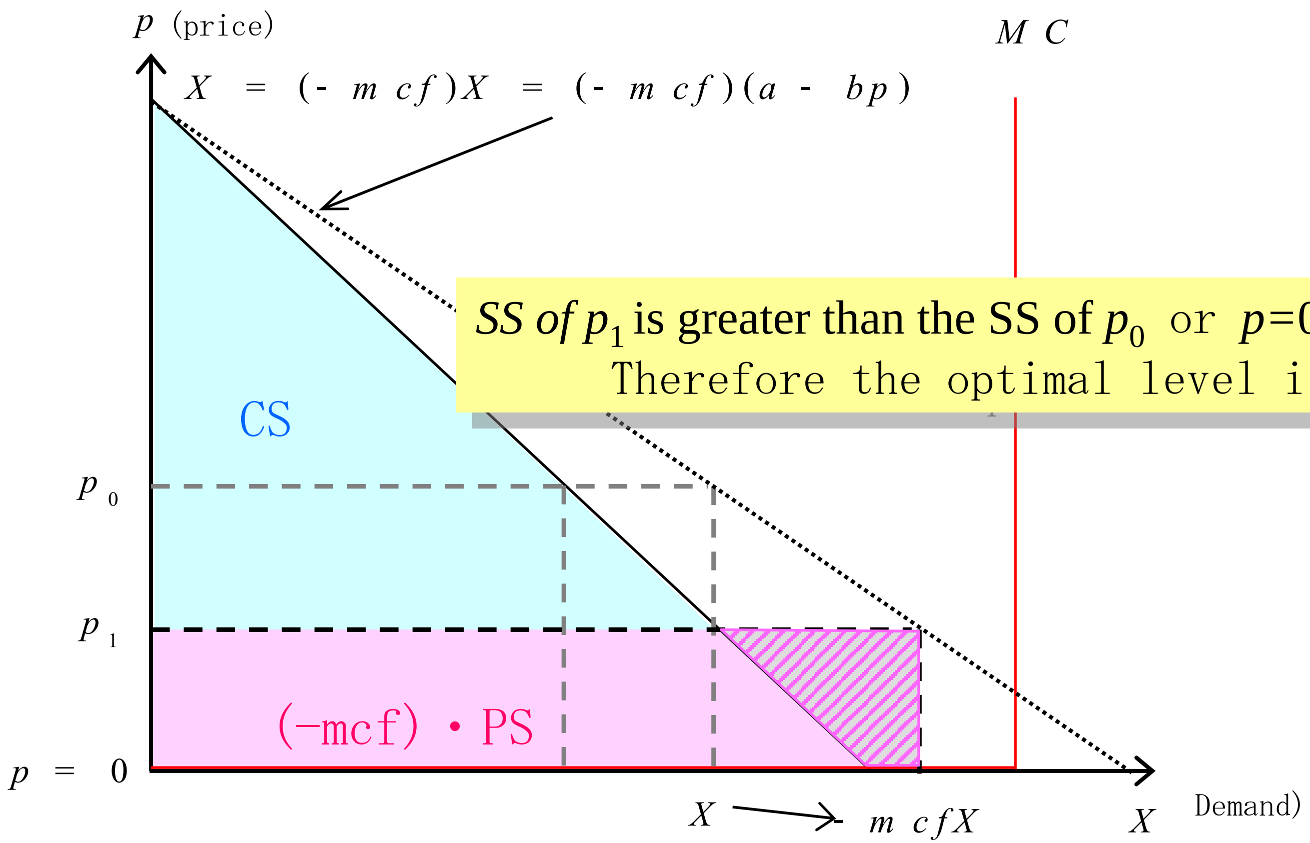
Take into Accounts MCF 3

SS of $p=0 =$ CS



Take into Accounts MCF 4

Max *SS CSp*・(*-mcf*)・*X*



SS of p_1 is greater than the SS of p_0 or p=0 by area
Therefore the optimal level is not zero

Take into Accounts MCF 5

The price level to maximize SS=(*CS+p*・*(-mcf)*・*X+contant)*

is such that the differentiation of SS with respect to p is zero

the differentiation of *CS with respect to p*

=－*X*（the area of *B in Fig. of* p.6 with *f*=0）

the differentiation of *p*(-*mcf*)*X with respect to p*

= (-*mcf*) *X*-*p* (-*mcf*) *b*

= (-*mcf*) (*X*-*pb*) （assume *X*=*a*-*bp*）

= the area of (*B*-*A*) *in Fig. of* p.6 with *f*=0）multiplied by (-*mcf*)

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Take into Accounts MCF 1

differentiation of SS = differentiation of *CS*
+ differentiation of *p(-mcf)X*
= *-X* +*(-mcf) (X-pb)= 0*

i.e. $p(-mcf)b=(-mcf-1)X$ （1）

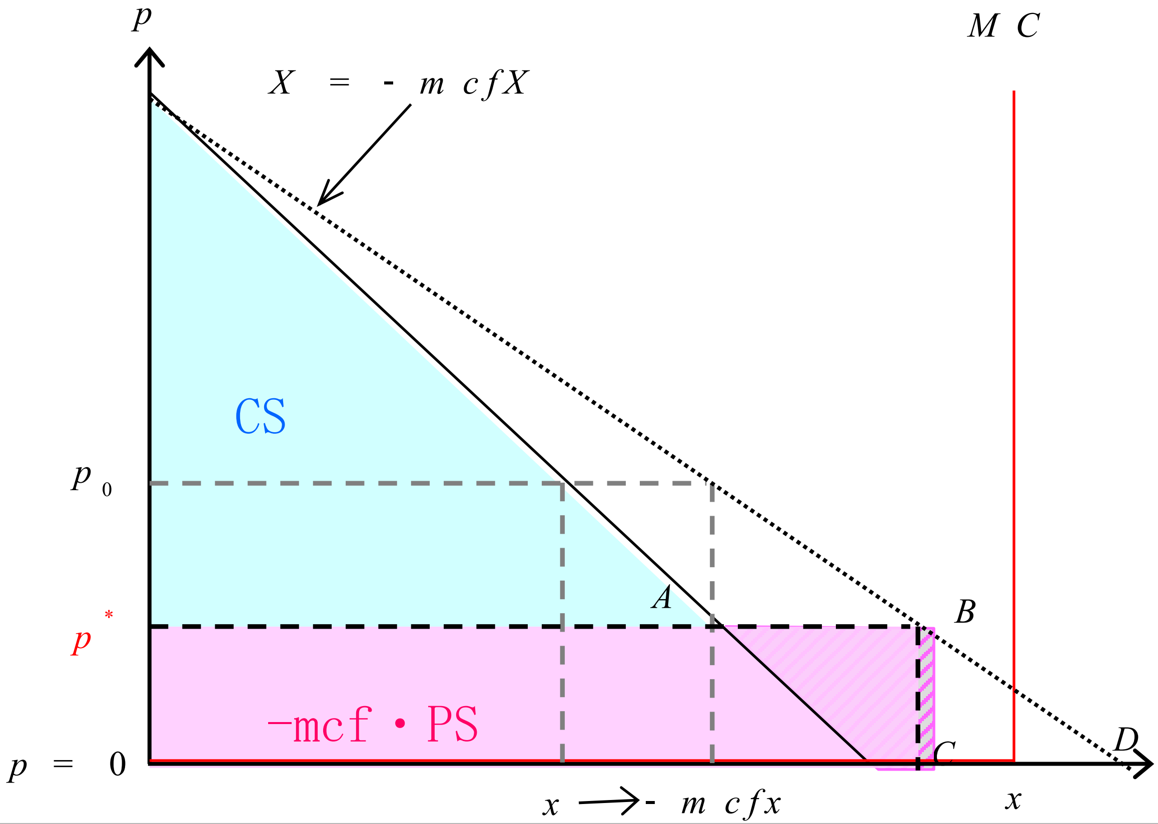
RHS of （1） =(-*mcf*-1)*X* =*AB* （length *AB* of in Fig. p.22）

As (-*mcf)X*=(-*mcf)(a-bp)* =(-*mcf)a-b(-mcf)p*

LHS of （1） ＝*p(-mcf)b*=(-*mcf)a*-(-*mcf)X*
=*CD* （length of *CD* in Fig.p.22 ）

Take into Accounts MCF 7

CD＝−*mcfa*+*mcfX*＝−*pmcfb*＝(-*mcf*-1)*X*＝AB



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Formula for optimal p (no) fuel tax

- *The optimal value of p is such that*

marginal cost of road price mcp

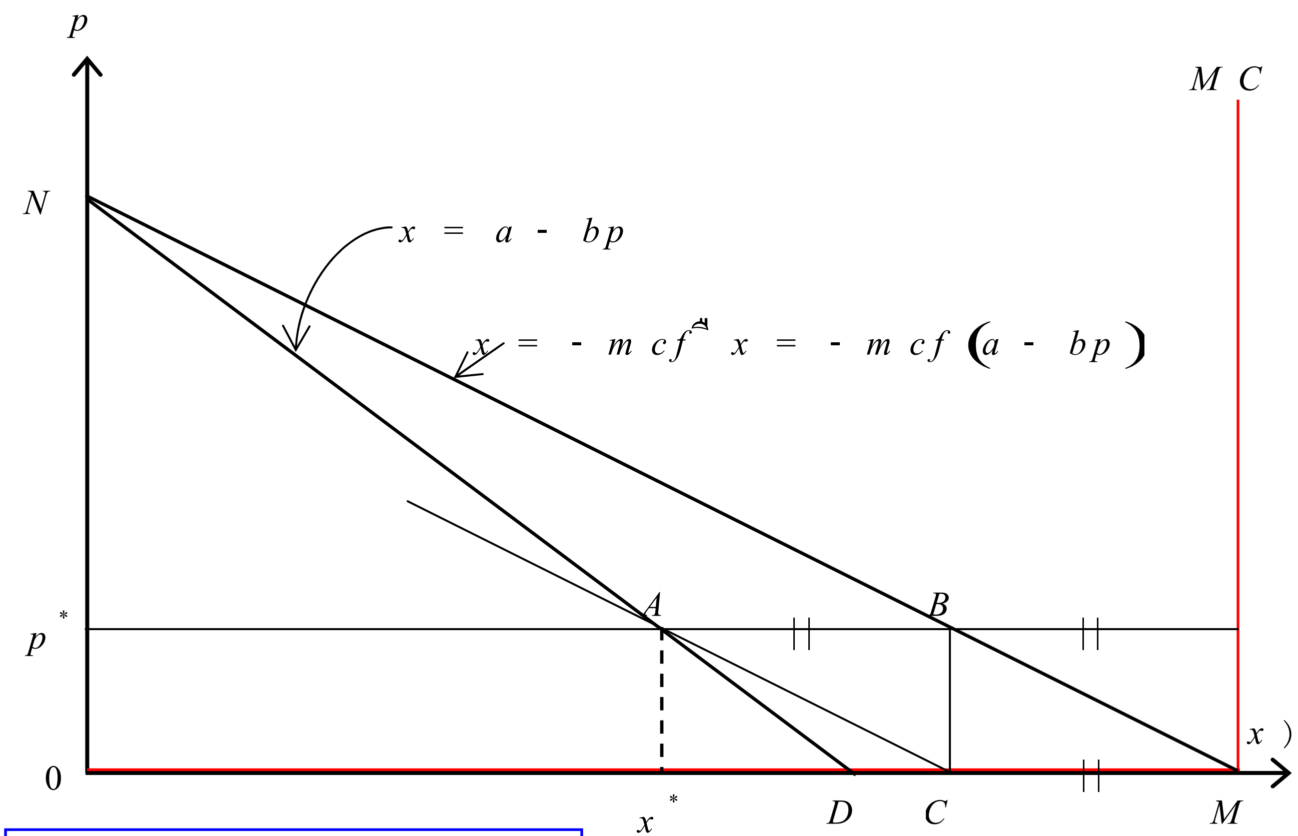
$= mcf$ *marginal cost of funding tax*

$$\partial SS / \partial p = \partial CS / \partial p + (-mcf)(\partial(pX) / \partial p)$$

$$= -X + (-mcp)(X + p\partial X / \partial p) = 0$$

$$mcp \equiv \frac{-X}{X + p(\partial X / \partial p)} = \frac{-1}{1 - \left|\varepsilon_p\right|} = mcf$$

Optimal price level 1



$$AB = (-mcf - 1)x^*$$

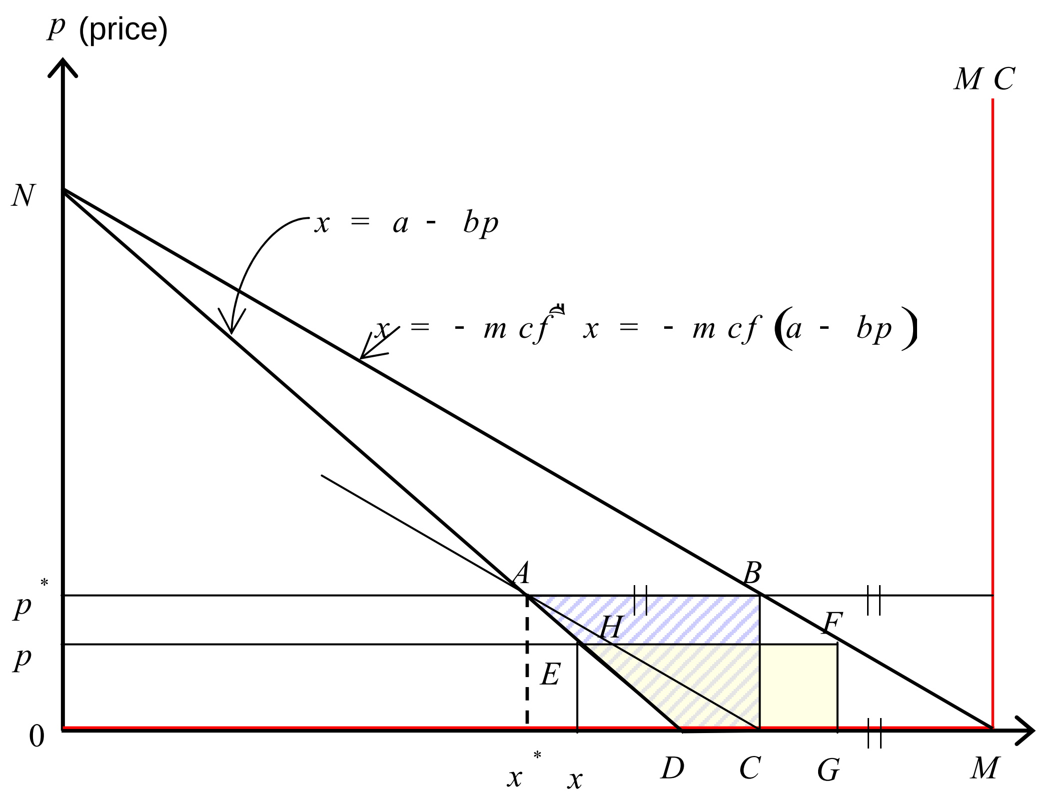
$$CM = -mcf^{-1} a - (-mcf)^{-1} x^*$$

$$= -mcf^{-1} a + mcf(a - bp^*)$$

$$= -mcf^{-1} b p^*$$

$$\Rightarrow \quad -mcf^{-1} b p^* = (-mcf - 1)x$$

Optimal price level 2



SS for p* − △N0D

＝□ABCD＝△ABC＋△ACD

SS for p − △N0D

＝□EFGD＝□HFGC＋□EHCD

△ABC＞□HFGC

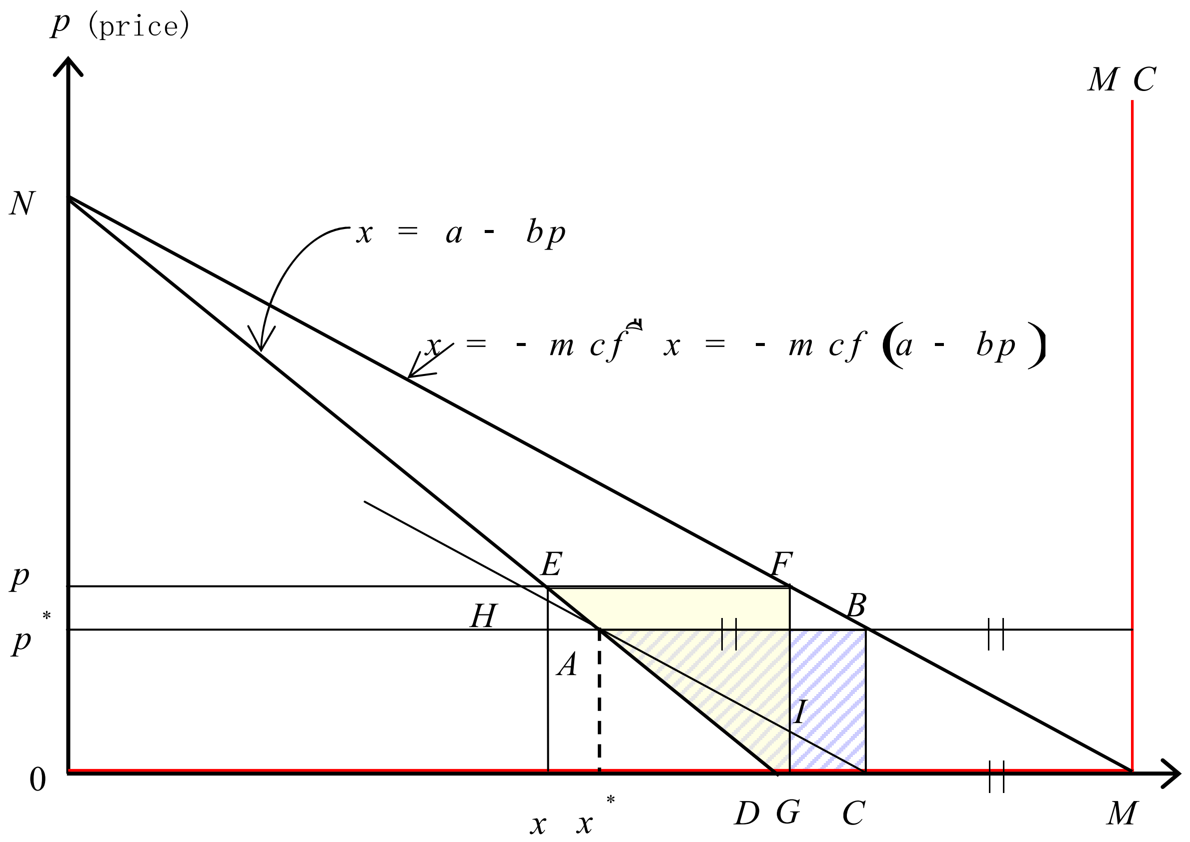
（AB＝HF、BC＞FG and parallels.
make □EFGC parallel movement to AB.

Then it is included into △ABC）

△ACD＞□EHCD

Therefore △ABCD＞□EFGD

Optimal price level 3



SS for p* − △N0D

= □ABCD = △ABC + △ACP

SS for p − △N0D

= □EFGD = □EFIA + □AIGD

△ABC = △HFI = □EFIA + △HEA

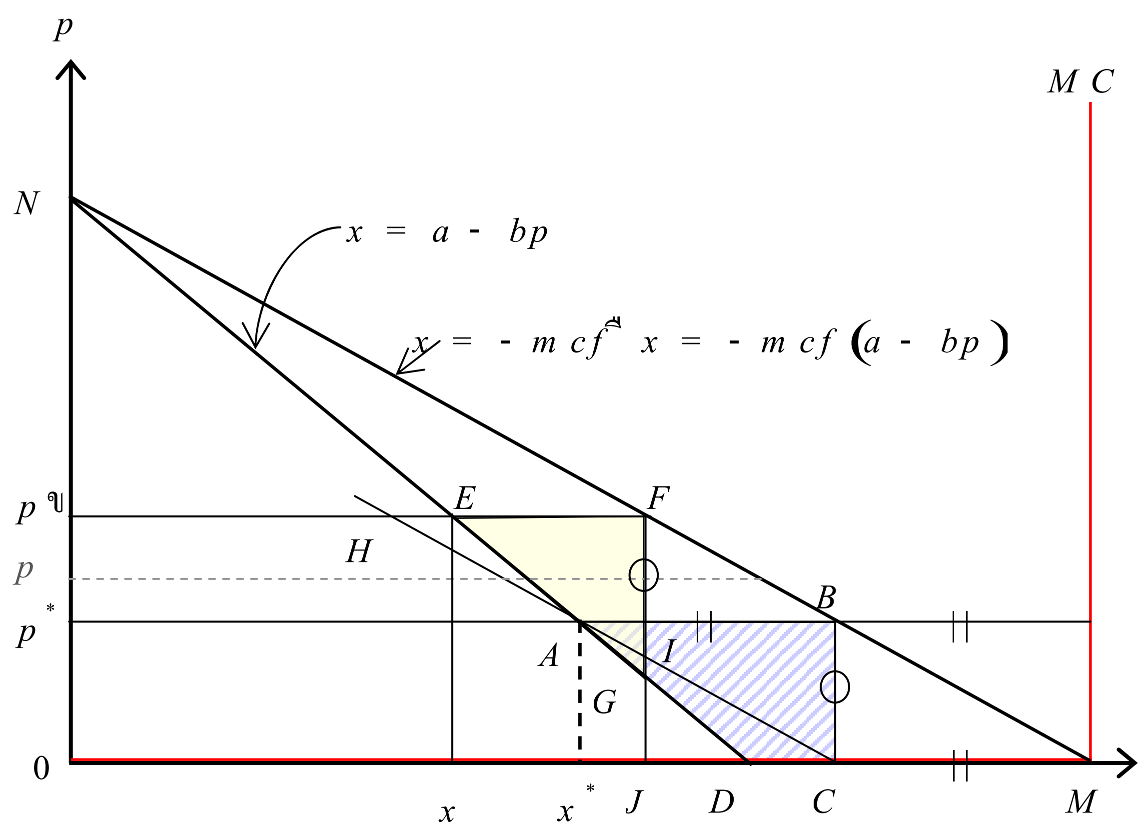
> □HFIA

△ACD = □AIGD + △ICG

> □AIGD

Therefore △ABCD > □EFGD

Optimal price level 4



SS for p^* − △N0D

= □ABCD = △ABC + △ACD

SS for p' − △N0D

= △EFG − △GDJ

= □EFIA + △AIG − △GDJ

△ABC = △HFI > □EFIA

△ACD > △AIG

Therefore △ABCD > △EFG − △GDJ

Formula for Optimal P
(with fuel tax and no parallel roads)

$$\max SS = \left(CS + PS - C + S + mcf \cdot S\right)$$

$$= CS + \left((p + (f / l))X - C + S\right) + mcf \cdot S$$

$$= CS + mcf\left(C - (p + (f / l))X\right)$$

$$= CS - mcf \cdot (p + (f / l))X + mcf \cdot C$$

$$= CS + (p + (f / l)) \cdot (-mcf) \cdot X + const$$

Formula for Optimal P
(with fuel tax and no parallel roads)

$$\partial SS/\partial p = \partial CS/\partial p - mcf\,\partial PS/\partial p$$
$$= -X - mcfX - mcf(p+(f/l))\partial X/\partial p$$
$$= -(1+mcf)X - mcf(p+(f/l))\partial X/\partial p$$
$$= -(1+mcf)X - mcf(p+(f/l))(\partial X/\partial C)(C/X)(X/C)\partial C/\partial p$$
$$= -(1+mcf)X + mcf((p+(f/l))/(p+(f/l)+cc))|\varepsilon| X$$
$$= [mcf/(p+(f/l)+mc)]X\{[|\varepsilon| - ((1+mcf)/mcf)](p+(f/l)) - ((1+mcf)/mcf)cc]\} = 0$$

$$p = \frac{[(1+mcf)/mcf]cc}{|\varepsilon| - [(1+mcf)/mcf)]} - \frac{f}{l}$$

$|\varepsilon| = |(\partial X/\partial C)(C/X)|$ full toll road price elasticity of parallel road trafficfuel

$C = p + (f/l) + cc =$ toll+fuel tax+vehicle operation cost

Formula for Optimal P
(with fuel tax and no parallel roads)

$$p = \frac{[(1+mcf)/mcf][cc + (f/l_O)(X_O/X)|\varepsilon_{HO}|]}{|\varepsilon| - [(1+mcf)/mcf)]} - \frac{f}{l}$$

X_O : Trafiic volume of parallel roads

l_O: fuel efficioency on parallel roads

$|\varepsilon_{HO}| = |(\partial X_O / \partial C)(C / X_O)|$: full toll road price elasticity of parallel road traffic

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Case Study

- Fuel tax f= 60\/l
- Fuel cost before tax g=40\/l
- Fuel efficiency on toll roads l=12vkm/l
- Fuel efficiency on ordinary roads l_0=8vkm/l
- Car price h=10\/vkm
- Time for 1 km run on toll roads t=0.75min/vkm
- Time for 1 km run on ordinary roads t_0=2 min/vkm
- Value of time wage after tax w=40\/min
- mcf=-1.1, -1.15, -1.2

Case Study

Toll road full price C=toll p+fuel tax (f/l)
+operation cost before tax cc(\/vkm
toll 48.33(*\/vkm*）

Fuel tax(f/l)60/12=5\/vkm

operation cost before tax cc(\/vkm）
*fuel cost before tax*40/12
car price10time cost40x0.75=43.33Yen/veh-km

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Case Study

	Average daily traffic volume on toll road x (vehicles/day)	Average daily traffic volume on parallel road x_O (vehicles/day)	Full toll road price elasticity of toll road traffic ε	Full toll road price cross-elasticity of parallel road tarafiic ε_{HO}
A-1 (18.9km)	**26000**	**38000**	0.3	0.1
A-2 (12.7km	**9100**	**26700**	0.4	0.1
A-3 (21.5km)	**4800**	**18100**	0.6	0.2
S-1 (11.3km)	**22900**	**0**	0.3	0.0
S-2 (16.1km)	**14100**	**0**	0.2	0.0

A-1 mcf=-1.1

$$p = \frac{[(1+mcf)/mcf][cc+(f/l_O)(X_O/X)|\varepsilon_{HO}|]}{|\varepsilon| - [(1+mcf)/mcf)]} - \frac{f}{l}$$

$$p = \frac{\left[\left(1+(-1.10)\right)/(-1.10)\right]\times\left[43.33+(60/8)\times(38000/26000)\times(0.1)\right]}{0.3-\left[\left(1+(-1.10)/(-1.10)\right)\right]} - \frac{60}{12}$$

$$= 15.0\left(\text{yen}/km\right)$$

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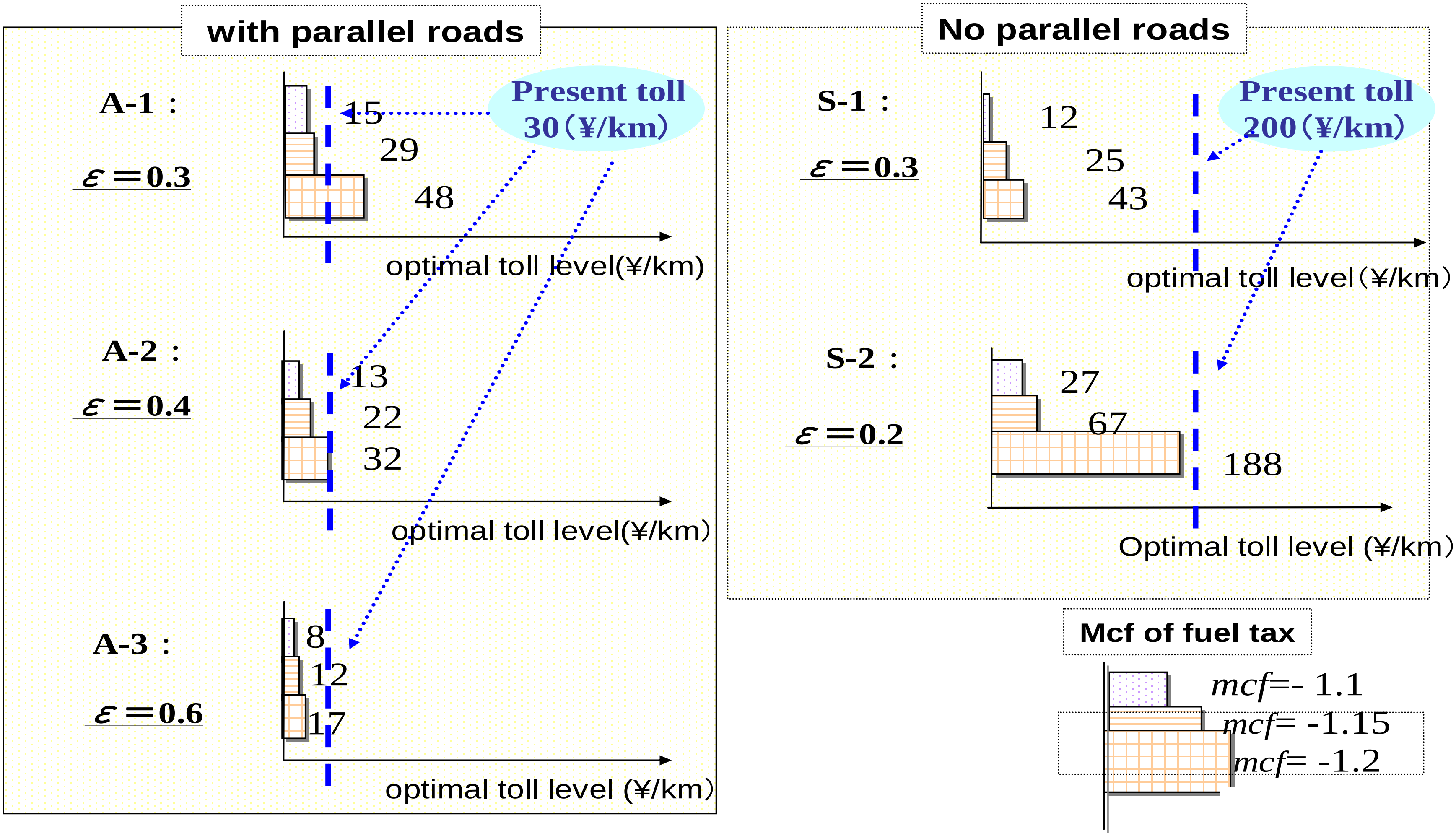
OPTIMAL TOLL LEVEL



Concluding Remarks

1. This study shows the formula to calculate the optimal toll level based on the efficiency principle taking into accounts the marginal cost of fuel tax
2. Applying it to the several toll road section in Japan, it shows the present level is much higher than the optimal level for almost all cases
3. So it can be said that it is recommended to lower the present toll level except for congested roads
4. But the zero price level is not recommendable when takes into accounts the marginal cost of that funds to construct the roads

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